Higher

Mathematics

Vectors

Examples

Contents

3 Magnitude

EF

1. Given $\underline{\boldsymbol{u}} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$, find $|\underline{\boldsymbol{u}}|$.

3 Magnitude EF

2. Find the length of $\underline{\boldsymbol{a}} = \begin{pmatrix} -\sqrt{5} \\ 6 \\ 3 \end{pmatrix}$.

3 Magnitude

EF

Distance in Three Dimensions

Find the distance between the points $(-1,4,1)$ and $(0,5,-7)$.

5 Addition and Subtraction of Vectors

EF

Using Components

1. Given $\underline{\boldsymbol{u}} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ and $\underline{\boldsymbol{v}} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$, calculate $\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}} - \underline{\boldsymbol{v}}$.

5 Addition and Subtraction of Vectors

EF

Using Components

2. Given $\underline{p} = \begin{pmatrix} 4 \\ \frac{3}{2} \\ 3 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} -1 \\ 3 \\ -\frac{6}{5} \end{pmatrix}$, calculate $\underline{p} - \underline{q}$ and $\underline{q} + \underline{p}$.

6 Multiplication by a Scalar EF

1. Given $\underline{\boldsymbol{v}} = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$, find $3\underline{\boldsymbol{v}}$.

6 Multiplication by a Scalar EF

2. Given $\underline{\boldsymbol{r}} = \begin{pmatrix} -6 \\ 3 \\ 1 \end{pmatrix}$, find $-4\underline{\boldsymbol{r}}$.

7 Position Vectors

EF

R is the point $(2, -2, 3)$ and S is the point $(4, 6, -1)$. Find $\overrightarrow{RS}$.

9 Collinearity EF

A is the point $(1, -2, 5)$, B$(8, -5, 9)$ and C$(22, -11, 17)$.

Show that A, B and C are collinear.

10 Dividing Lines in a Ratio

EF

1. P is the point $(-2, 4, -1)$ and R is the point $(8, -1, 19)$.

   The point T divides PR in the ratio $2:3$. Find the coordinates of T.

10 Dividing Lines in a Ratio

EF

Using the Section Formula

2. P is the point $(-2, 4, -1)$ and R is the point $(8, -1, 19)$.

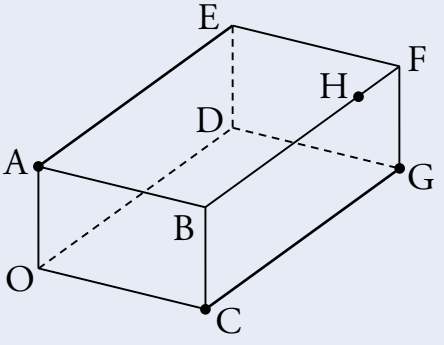
   The point T divides PR in the ratio $2:3$. Find the coordinates of T.

10 Dividing Lines in a Ratio

EF

Further Examples

3. The cuboid OABCDEFG is shown in the diagram.



The point A has coordinates $(0,0,5)$, C$(8,0,0)$ and G$(8,12,0)$. The point H divides BF in the ratio $4:1$. Find the coordinates of H.

10 Dividing Lines in a Ratio EF

Further Examples

4. The points $P(6,1,-3)$, $Q(8,-3,1)$ and $R(9,-5,3)$ are collinear. Find the ratio in which Q divides PR.

10 Dividing Lines in a Ratio EF

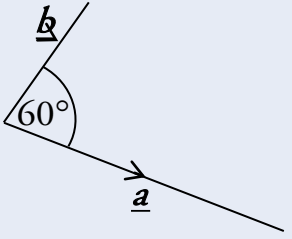
Further Examples

5. The points $A(7,-4,-4)$, $B(13,5,-7)$ and C are collinear. Given that B divides AC in the ratio $3:2$, find the coordinates of C.

11 The Scalar Product

EF

1. Two vectors, $\underline{a}$ and $\underline{b}$ have magnitudes 7 and 3 units respectively and are at an angle of 60° to each other as shown below.

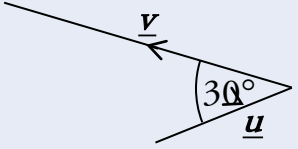


What is the value of $\underline{a}.\underline{b}$?

11 The Scalar Product

EF

2. The vector $\underline{\boldsymbol{u}}$ has magnitude k and $\underline{\boldsymbol{v}}$ is twice as long as $\underline{\boldsymbol{u}}$. The angle between $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$ is $30°$, as shown below.



Find an expression for $\underline{\boldsymbol{u}}.\underline{\boldsymbol{v}}$ in terms of k.

11 The Scalar Product

EF

The Component Form of the Scalar Product

3. Find $\underline{p}.\underline{q}$, given that $\underline{p}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$ and $\underline{q}=\begin{pmatrix}2\\2\\3\end{pmatrix}$.

11 The Scalar Product

EF

The Component Form of the Scalar Product

4. If A is the point $(2, 3, 9)$, B$(1, 4, -2)$ and C$(-1, 3, -6)$, calculate $\overrightarrow{AB}.\overrightarrow{AC}$.

12 The Angle Between Vectors

EF

1. Calculate the angle θ between vectors $\underline{p} = 3\underline{i} + 4\underline{j} - 2\underline{k}$ and $\underline{q} = 4\underline{i} + \underline{j} + 3\underline{k}$.

12 The Angle Between Vectors

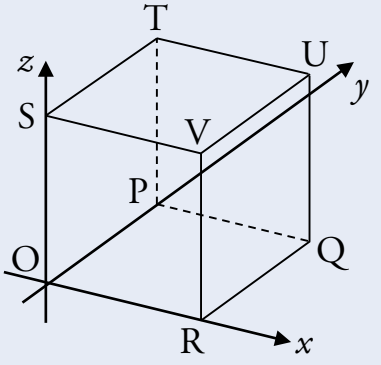
EF

2. K is the point $(1, -7, 2)$, L$(-3, 3, 4)$ and M$(2, 5, 1)$. Find $\hat{\text{KLM}}$.

12 The Angle Between Vectors

EF

3. The diagram below shows the cube OPQRSTUV.



The point R has coordinates $(4,0,0)$.

(a) Write down the coordinates of T and U.

(b) Find the components of $\overrightarrow{RT}$ and $\overrightarrow{RU}$.

(c) Calculate the size of angle TRU.

13 Perpendicular Vectors

EF

1. Two vectors are defined as $\underline{\boldsymbol{a}} = 4\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} - 5\underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{b}} = 2\underline{\boldsymbol{i}} + \underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$.

   Show that $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ are perpendicular.

13 Perpendicular Vectors

EF

2. $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ a \\ 7 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} 2 \\ -3 \\ a \end{pmatrix}$ where a is a constant.

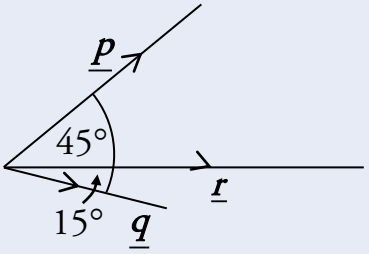
Given that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are perpendicular, find the value of a.

14 Properties of the Scalar Product

EF

1. In the diagram, $|\underline{p}| = 3$, $|\underline{r}| = 4$ and $|\underline{q}| = 2$.

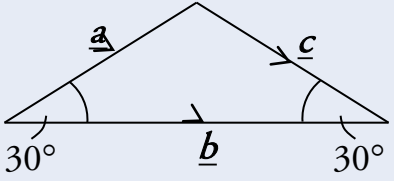
   Calculate $\underline{p}.(\underline{q} + \underline{r})$.



14 Properties of the Scalar Product

EF

2. In the diagram below $|\underline{a}| = |\underline{c}| = 2$ and $|\underline{b}| = 2\sqrt{3}$.



Calculate $\underline{a}.(\underline{a} + \underline{b} + \underline{c})$.