Higher

Mathematics

Polynomials and Quadratics

Examples

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1 Quadratics

EF

1. Find the roots of $x^2 - 2x - 3 = 0$.

1 Quadratics

EF

2. Solve $x^2 + 8x + 16 = 0$.

1 Quadratics

EF

3. Find the roots of $x^2 + 4x - 1 = 0$.

2 The Discriminant

EF

1. Find the nature of the roots of $9x^2 + 24x + 16 = 0$.

2 The Discriminant

EF

2. Find the values of q such that $6x^2 + 12x + q = 0$ has real roots.

2 The Discriminant EF

3. Find the range of values of k for which the equation $kx^2 + 2x - 7 = 0$ has no real roots.

2 The Discriminant EF

4. Show that $(2k+4)x^2+(3k+2)x+(k-2)=0$ has real roots for all real values of k.

3 Completing the Square

EF

1. Write $y = x^2 + 6x - 5$ in the form $y = (x + p)^2 + q$.

3 Completing the Square

EF

2. Write $x^2 + 3x - 4$ in the form $(x + p)^2 + q$.

3 Completing the Square EF

3. Write $y = x^2 + 8x - 3$ in the form $y = (x + a)^2 + b$ and then state:

   (i) the axis of symmetry, and

   (ii) the minimum turning point of the parabola with this equation.

3 Completing the Square EF

4. A parabola has equation $y = 4x^2 - 12x + 7$.

   (a) Express the equation in the form $y = (x + a)^2 + b$.

   (b) State the turning point of the parabola and its nature.

4 Sketching Parabolas EF

1. Sketch the graph of $y = x^2 - 8x + 7$.

4 Sketching Parabolas EF

2. Sketch the parabola with equation $y = -x^2 - 6x - 9$.

4 Sketching Parabolas EF

3. Sketch the curve with equation $y = 2x^2 - 8x + 13$.

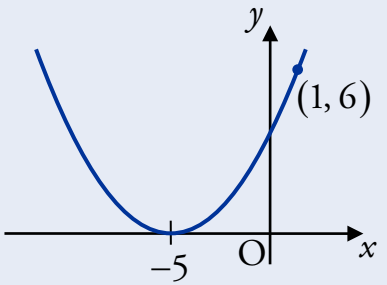
5 Determining the Equation of a Parabola RC

1. A parabola passes through the points $(1, 0)$, $(5, 0)$ and $(0, 3)$.

   Find the equation of the parabola.

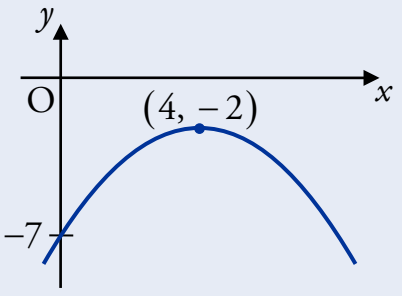
5 Determining the Equation of a Parabola RC

2. Find the equation of the parabola shown below.



5 Determining the Equation of a Parabola RC

3. Find the equation of the parabola shown below.



6 Solving Quadratic Inequalities

RC

1. Solve $x^2 + x - 12 < 0$.

6 Solving Quadratic Inequalities RC

2. Find the values of x for which $6+7x-3x^2 \geq 0$.

6 Solving Quadratic Inequalities RC

3. Solve $2x^2 - 5x - 3 > 0$.

6 Solving Quadratic Inequalities RC

4. Find the range of values of x for which the curve $y = \frac{1}{3}x^3 + 2x^2 - 5x + 3$ is strictly increasing.

6 Solving Quadratic Inequalities RC

5. Find the values of q for which $x^2 + (q-4)x + \frac{1}{2}q = 0$ has no real roots.

7 Intersections of Lines and Parabolas RC

1. Show that the line $y = 5x - 2$ is a tangent to the parabola $y = 2x^2 + x$ and find the point of contact.

7 Intersections of Lines and Parabolas RC

2. Find the equation of the tangent to $y = x^2 + 1$ that has gradient 3.

9 Synthetic Division RC

1. Given $f(x)=x^3+x^2-22x-40$, evaluate $f(-2)$ using synthetic division.

9 Synthetic Division

RC

2. Show that $x-4$ is a factor of $2x^4-9x^3+5x^2-3x-4$.

9 Synthetic Division RC

3. Given $f(x) = x^3 - 37x + 84$, show that $x = -7$ is a root of $f(x) = 0$, and hence fully factorise $f(x)$.

9 Synthetic Division RC

4. Show that $x = -5$ is a root of $2x^3 + 7x^2 - 9x + 30 = 0$, and hence fully factorise the cubic.

9 Synthetic Division

RC

Using synthetic division to factorise

5. Fully factorise $2x^3 + 5x^2 - 28x - 15$.

9 Synthetic Division

RC

Using synthetic division to solve equations

6. Find the solutions of $2x^3 - 15x^2 + 16x + 12 = 0$.

9 Synthetic Division

RC

The Factor Theorem and Remainder Theorem

7. Find the quotient and remainder when $f(x) = 4x^3 + x^2 - x - 1$ is divided by $x + 1$, and express $f(x)$ as $(x+1)q(x) + f(h)$.

10 Finding Unknown Coefficients RC

1. Given that $x-3$ is a factor of $x^3 - x^2 + px + 24$, find the value of p.

10 Finding Unknown Coefficients RC

2. When $f(x) = px^3 + qx^2 - 17x + 4q$ is divided by $x - 2$, the remainder is 6, and $x - 1$ is a factor of $f(x)$.

   Find the values of p and q.

11 Finding Intersections of Curves RC

1. Find the points of intersection of the line $y = 4x - 4$ and the parabola $y = 2x^2 - 2x - 12$.

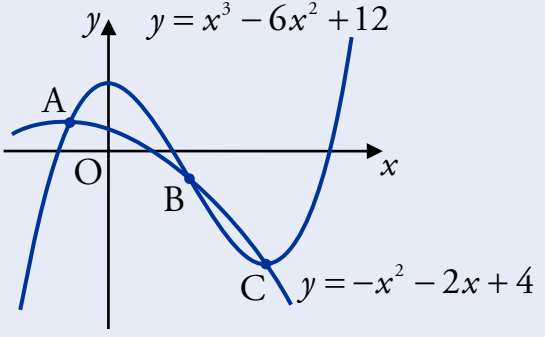
11 Finding Intersections of Curves RC

2. Find the coordinates of the points of intersection of the cubic $y = x^3 - 9x^2 + 20x - 10$ and the line $y = -3x + 5$.

11 Finding Intersections of Curves

RC

3. The curves $y = -x^2 - 2x + 4$ and $y = x^3 - 6x^2 + 12$ are shown below.



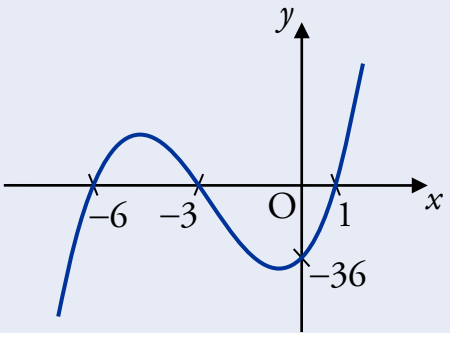
Find the x-coordinates of A, B and C, where the curves intersect.

11 Finding Intersections of Curves RC

4. Find the x-coordinates of the points where the curves $y = 2x^3 - 3x^2 - 10$ and $y = 3x^3 - 10x^2 + 7x + 5$ intersect.

12 Determining the Equation of a Curve RC

1. Find the equation of the cubic shown in the diagram below.



12 Determining the Equation of a Curve RC

2. Find the equation of the cubic shown in the diagram below.