Higher

Mathematics

Integration

Examples

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1. Find $\int x^2 \, dx$.

1 Indefinite Integrals

RC

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1 Indefinite Integrals

RC

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1 Indefinite Integrals RC

Integrating terms with coefficients

4. Find $\int 6x^3 \, dx$.

1 Indefinite Integrals RC

Integrating terms with coefficients

5. Find $\int 4x^{-\frac{3}{2}}\,dx$.

1 Indefinite Integrals RC

Other variables

6. Find $\int 2p^{-5}\ dp$.

1 Indefinite Integrals RC

Other variables

7. Find $\int p \; dx$.

1 Indefinite Integrals

RC

Integrating several terms

8. Find $\int \left(3x^2 - 2x^{\frac{1}{2}}\right) dx$.

1 Indefinite Integrals

RC

Integrating several terms

9. Find $\int \left(4x^{-\frac{5}{8}} + 3x + 7\right) dx$.

2 Preparing to Integrate RC

1. Find $\int \frac{dx}{x^2}$ for $x \neq 0$.

2 Preparing to Integrate RC

2. Find $\int \frac{dx}{\sqrt{x}}$ for $x > 0$.

2 Preparing to Integrate RC

3. Find $\int \frac{7}{2p^2}\,dp$ where $p \neq 0$.

2 Preparing to Integrate RC

4. Find $\int \frac{3x^5 - 5x}{4}\, dx$.

3 Differential Equations A

1. The graph of $y = f(x)$ passes through the point $(3, -4)$.

   If $\frac{dy}{dx} = x^2 - 5$, express y in terms of x.

3 Differential Equations A

2. The function f, defined on a suitable domain, is such that $f'(x) = x^2 + \frac{1}{x^2} + \frac{2}{3}$.

   Given that $f(1) = 4$, find a formula for $f(x)$ in terms of x.

4 Definite Integrals

RC

1. Find $\int_1^3 5x^2 \, dx$.

4 Definite Integrals RC

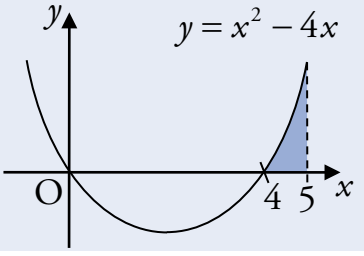
2. Find $\int_0^2 \left(x^3 + 3x^2\right) dx$.

4 Definite Integrals RC

3. Find $\int_{-1}^{4} \frac{4}{x^3}\,dx$.

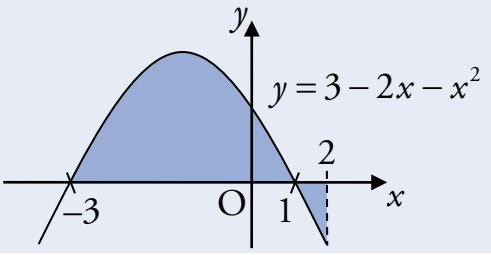
5 Geometric Interpretation of Integration A

1. The graph of $y = x^2 - 4x$ is shown below. Calculate the shaded area.



5 Geometric Interpretation of Integration A

2. Calculate the shaded area shown in the diagram below.



5 Geometric Interpretation of Integration A

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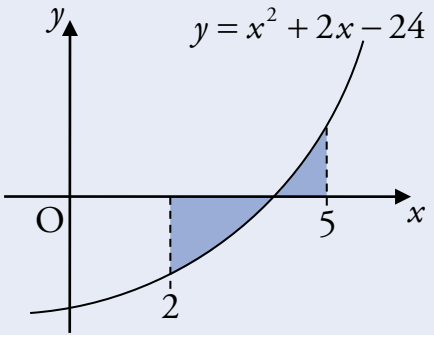
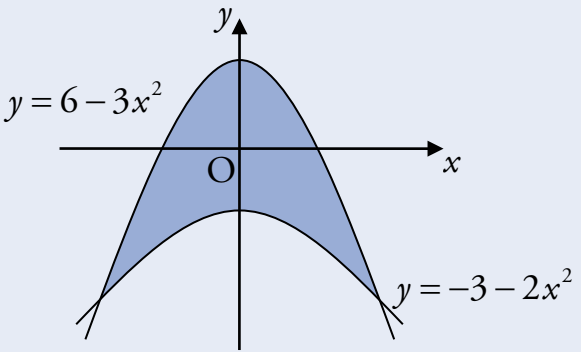


6 Areas between Curves

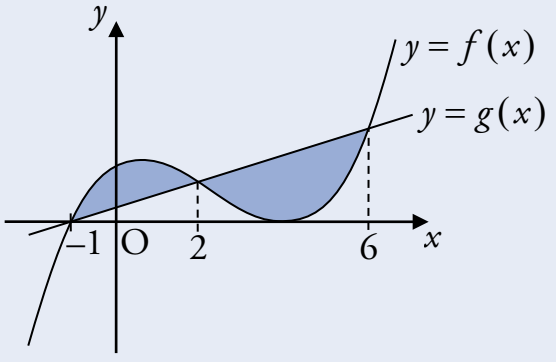
A

1. Calculate the shaded area enclosed by the curves with equations $y = 6 - 3x^2$ and $y = -3 - 2x^2$.



6 Areas between Curves A

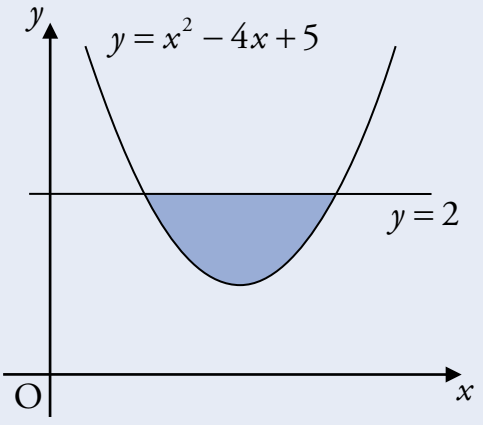
2. Two functions are defined for $x \in \mathbb{R}$ by $f(x) = x^3 - 7x^2 + 8x + 16$ and $g(x) = 4x + 4$. The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



Calculate the shaded area.

6 Areas between Curves A

3. A trough is 2 metres long. A cross-section of the trough is shown below.

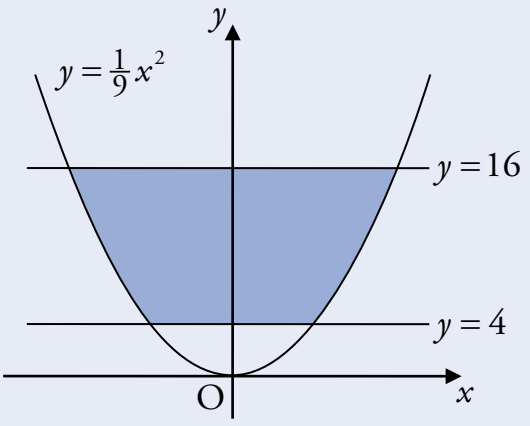


The cross-section is part of the parabola with equation $y = x^2 - 4x + 5$.

Find the volume of the trough.

7 Integrating along the *y*-axis RC

The curve with equation $y = \frac{1}{9}x^2$ is shown in the diagram below.



Calculate the shaded area which lies between $y = 4$ and $y = 16$.

8 Integrating sin*x* and cos*x* RC

1. Find $\int (5\sin x + 2\cos x)\, dx$.

8 Integrating sin*x* and cos*x* RC

2. Find $\int_0^{\frac{\pi}{4}} (4\cos x + 2\sin x)\ dx$.

8 Integrating sin*x* and cos*x* RC

3. Find the value of $\int_0^4 \frac{1}{2}\sin x \ dx$.

9 A Special Integral

RC

1. Find $\int (x+4)^7 \, dx$.

9 A Special Integral

RC

2. Find $\int (2x+3)^2 \, dx$.

9 A Special Integral RC

3. Find $\int \frac{1}{\sqrt[3]{5x+9}}\,dx$ where $x \neq -\frac{9}{5}$.

9 A Special Integral

RC

4. Evaluate $\int_0^3 \sqrt{3x+4}\ dx$ where $x \geq -\frac{4}{3}$.

9 A Special Integral

RC

Using Differentiation to Integrate

5. (a) Differentiate $y = \dfrac{5}{(3x-1)^4}$ with respect to x.

(b) Hence, or otherwise, find $\displaystyle\int \frac{1}{(3x-1)^5}\,dx$.

9 A Special Integral

RC

Using Differentiation to Integrate

6. (a) Differentiate $y = \dfrac{1}{\left(x^3 - 1\right)^5}$ with respect to x.

(b) Hence, find $\displaystyle\int \frac{x^2}{\left(x^3 - 1\right)^6}\, dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

1. Find $\int \sin(4x+1)\,dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

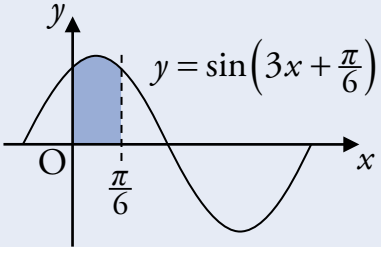
2. Find $\int \cos\left(\frac{3}{2}x + \frac{\pi}{5}\right) dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

3. Find the value of $\int_0^1 \cos(2x-5)\,dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

4. Find the area enclosed be the graph of $y = \sin\left(3x + \frac{\pi}{6}\right)$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{6}$.



10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

5. Find $\int 2\cos\left(\frac{1}{2}x - 3\right)\,dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

6. Find $\int 5\cos(2x) + \sin\left(x - \sqrt{3}\right)\, dx$.

10 Integrating sin(*ax* + *b*) and cos(*ax* + *b*) RC

7. (a) Differentiate $\frac{1}{\cos x}$ with respect to x.

   (b) Hence find $\int \frac{\tan x}{\cos x}\, dx$.