Higher

Mathematics

Exponentials and Logarithms

Examples

Contents

1 Exponentials

EF

1. The otter population on an island increases by 16% per year. How many full years will it take the population to double?

1 Exponentials

EF

2. The efficiency of a machine decreases by 5% each year. When the efficiency drops below 75%, the machine needs to be serviced.

   After how many years will the machine need to be serviced?

2 Logarithms

EF

1. Write $5^3 = 125$ in logarithmic form.

2 Logarithms EF

2. Evaluate $\log_4 16$.

3 Laws of Logarithms EF

Rule 1

1. Simplify $\log_5 2 + \log_5 4$.

3 Laws of Logarithms

EF

Rule 2

2. Evaluate $\log_4 6 - \log_4 3$.

3 Laws of Logarithms

EF

Rule 3

3. Express $2\log_7 3$ in the form $\log_7 a$.

3 Laws of Logarithms

EF

4. Evaluate $\log_7 7 + \log_3 3$.

3 Laws of Logarithms

EF

Combining several log terms

5. Evaluate $\log_{12} 10 + \log_{12} 6 - \log_{12} 5$.

3 Laws of Logarithms

EF

Combining several log terms

6. Evaluate $\log_6 4 + 2\log_6 3$.

4 Exponentials and Logarithms to the Base *e*

EF

1. Calculate the value of $\log_e 8$.

4 Exponentials and Logarithms to the Base *e*

EF

2. Solve $\log_e x = 9$.

4 Exponentials and Logarithms to the Base *e* EF

3. Simplify $4\log_e(2e) - 3\log_e(3e)$ expressing your answer in the form $a + \log_e b - \log_e c$ where a, b and c are whole numbers.

5 Exponential and Logarithmic Equations

EF

1. Solve $\log_a 13 + \log_a x = \log_a 273$ for $x > 0$.

5 Exponential and Logarithmic Equations EF

2. Solve $\log_{11}(4x+3)-\log_{11}(2x-3)=1$ for $x>\frac{3}{2}$.

5 Exponential and Logarithmic Equations

EF

3. Solve $\log_a(2p+1)+\log_a(3p-10)=\log_a(11p)$ for $p>4$.

5 Exponential and Logarithmic Equations

EF

Dealing with Constants

4. Solve $\log_2 7 = \log_2 x + 3$ for $x > 0$.

5 Exponential and Logarithmic Equations

EF

Solving Equations with unknown Exponents

5. Solve $e^x = 7$.

5 Exponential and Logarithmic Equations

EF

Solving Equations with unknown Exponents

6. Solve $5^{3x+1} = 40$.

5 Exponential and Logarithmic Equations EF

Exponential Growth and Decay

7. The mass G grams of a radioactive sample after time t years is given by the formula $G = 100e^{-3t}$.

   (a) What is the initial mass of radioactive substance in the sample?

   (b) Find the half-life of the radioactive substance.

5 Exponential and Logarithmic Equations

EF

Exponential Growth and Decay

8. The world population, in billions, t years after 1950 is given by $P = 2{\cdot}54e^{0{\cdot}0178t}$.

   (a) What was the world population in 1950?

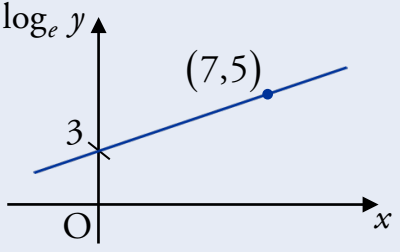
   (b) Find, to the nearest year, the time taken for the world population to double.

6 Graphing with Logarithmic Axes

EF

Relationships of the form $y = ab^x$

1. The relationship between two variables, x and y, is of the form $y = ab^x$, where a and b are constants. An experiment to test this relationship produced the data shown in the graph, where $\log_e y$ is plotted against x.



Find the values of a and b.

6 Graphing with Logarithmic Axes

EF

Relationships of the form $y = ab^x$

2. The results from an experiment were noted as follows:

x	1·30	2·00	2·30	2·80
$\log_e y$	2·04	2·56	2·78	3·14

The relationship between these data can be written in the form $y = ab^x$.

Find the values of a and b, and state the formula for y in terms of x.

6 Graphing with Logarithmic Axes

EF

Equations in the form $y = ax^b$

3. The results from an experiment were noted as follows:

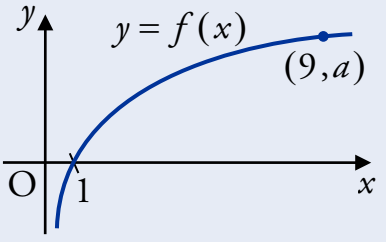
$\log_{10} x$	1·70	2·29	2·70	2·85
$\log_{10} y$	1·33	1·67	1·92	2·01

The relationship between these data can be written in the form $y = ax^b$.

Find the values of a and b, and state the formula for y in terms of x.

7 Graph Transformations EF

1. Shown below is the graph of $y = f(x)$ where $f(x) = \log_3 x$.



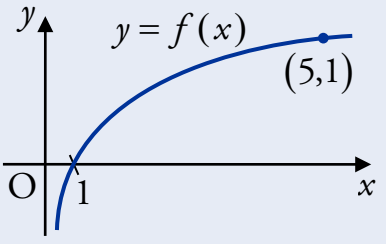
(a) State the value of a.

(b) Sketch the graph of $y = f(x+2)+1$.

7 Graph Transformations

EF

2. Shown below is part of the graph of $y = \log_5 x$.

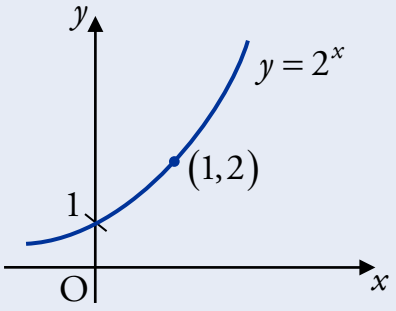


Sketch the graph of $y = \log_5\left(\frac{1}{x}\right)$.

7 Graph Transformations

EF

3. The diagram shows the graph of $y = 2^x$.



On separate diagrams, sketch the graphs of:

(a) $y = 2^{-x}$;

(b) $y = 2^{2-x}$.