Higher

Physics

HSN61200
Unit 1 Topic 2

Contents

Topic 2 – Kinematics

Introduction to Acceleration

The acceleration of an object is defined as its change in speed per unit time. A negative acceleration (ie when the velocity is decreasing) is sometimes called deceleration. Acceleration is usually given in units of ms^{-2}.

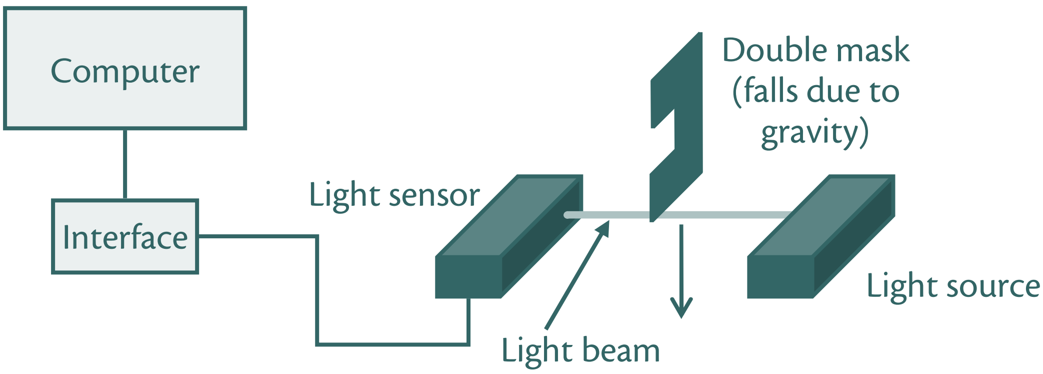
The acceleration of an object with initial speed u and final speed v over a time interval t is given as:

$$a = \frac{v-u}{t}$$

Measuring the Acceleration due to Gravity

Aim
To measure g, the acceleration due to gravity.

Procedure
The following setup is used in this experiment:



The double mask is used to interrupt the light beam and send a signal to the computer. The computer measures the initial speed u, the final speed v, and the time interval t. It then calculates the acceleration a using the formula:

$$a = \frac{v-u}{t}$$

where a is the acceleration due to gravity (g).

Results

Trial No	1	2	3	4	5	6	7	8	9	10	11
g measured	9.93	9.30	10.10	9.32	9.80	9.99	8.28	9.88	10.14	9.96	9.72

We can estimate g by finding the average of the readings:

$$g \text{ average} = \frac{\text{sum of } g\text{s}}{\text{number of trials}} = \frac{106.42}{11} = 9.67 \text{ ms}^{-2}$$

Since this value has been found empirically (by experiment) we must take uncertainties into account:

$$\text{random uncertainty} = \frac{\text{largest reading} - \text{smallest reading}}{\text{number of trials}} = \frac{10.14 - 8.28}{11} = 0.17$$

Therefore $g = (9.67 \pm 0.17)\ \text{ms}^{-2}$.

The Equations of Motion

There are three equations which can be used to work out properties of the motion of an object, known collectively as the equations of motion. These depend on there being a uniform (constant) acceleration, note however that this may be zero.

Deriving the Equations of Motion

We have already learned that the acceleration of an object is calculated as:

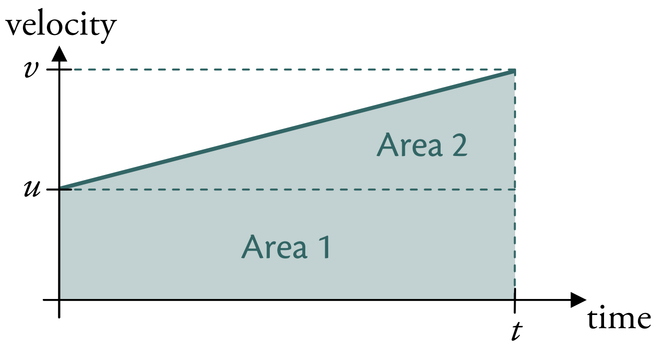
$$a = \frac{v - u}{t}$$

where v and u are the final and initial velocities respectively, and t is the time interval.

Rearranging this equation to find v gives the first equation of motion:

$$v = u + at$$

The displacement of an object can be calculated from its velocity-time graph. The displacement is equal in magnitude to the area enclosed by the graph and the time axis, as shown below.



$$\begin{aligned}\text{Total area} &= \text{Area 1} + \text{Area 2}\\ &= ut + \tfrac{1}{2}t \times (v - u) \qquad v = u + at \Rightarrow v - u = at\\ &= ut + \tfrac{1}{2}t \times at\\ &= ut + \tfrac{1}{2}at^2\end{aligned}$$

This gives us the second equation of motion:

$$s = ut + \tfrac{1}{2}at^2$$

Where s is the displacement, u is the initial speed, a is the acceleration, and t is the time interval.

Now if we take the first equation of motion and square it:

$$\begin{aligned} v^2 &= (u+at)^2 \\ &= (u+at)(u+at) \\ &= u^2 + 2atu + a^2t^2 \\ &= u^2 + 2a\left(ut + \tfrac{1}{2}at^2\right) \qquad s = ut + \tfrac{1}{2}at^2 \\ &= u^2 + 2as \end{aligned}$$

This gives us the third equation of motion:

$$v^2 = u^2 + 2as$$

Where u and v are the initial and final speeds respectively, a is the acceleration and s is the displacement.

We have therefore derived the three equations of motion:

$$v = u + at \qquad s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

Which equation to use depends on the particular problem and the information you are given in the problem. Sometimes it will be necessary to use more than one equation in a single problem.

Examples

1. A car initially travelling at 10ms^{-1} accelerates at 0.5ms^{-2} for 8 seconds. Find its speed after this time interval.

   First write down what we know:

   $$\begin{aligned} u &= 10 \text{ ms}^{-1} \\ a &= 0.5 \text{ ms}^{-2} \\ t &= 8 \text{ s} \\ v &= ? \text{ ms}^{-1} \end{aligned}$$

   Then work out what we are asked:

   $$\begin{aligned} v &= u + at \\ &= 10 + 0.5 \times 8 \\ &= 10 + 4 \\ &= 14 \text{ ms}^{-1} \end{aligned}$$

   So the final speed of the car is 14 ms^{-1}.

2. A hedgehog crossing a road has an initial speed of 0.3 ms^{-1} and accelerates uniformly at 0.04 ms^{-2}. If the hedgehog takes 10 seconds to cross the road, calculate the width of the road.

$$u = 0.3 \text{ ms}^{-1} \qquad a = 0.04 \text{ ms}^{-2} \qquad t = 10 \text{ s} \qquad s = ? \text{ m}$$

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ &= 0.3\times10 + \tfrac{1}{2}\times0.04\times10^2 \\ &= 3 + 2 \\ &= 5 \text{ m} \end{aligned}$$

So the width of the road is 5 metres.

3. A satellite has an initial speed of 500 ms^{-1}. The retro rockets are fired, causing a deceleration of 15 ms^{-2}. Calculate the distance over which the speed will fall to 200 ms^{-1}.

$$u = 500 \text{ ms}^{-1} \qquad a = -15 \text{ ms}^{-2} \qquad v = 200 \text{ ms}^{-1} \qquad s = ? \text{ m}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 2as &= v^2 - u^2 \\ s &= \frac{v^2 - u^2}{2a} \\ &= \frac{200^2 - 500^2}{2\times(-15)} \\ &= 7000 \text{ m} \end{aligned}$$

So the distance is 7 kilometres.

Two-Dimensional Motion

The equations of motion can be used to help solve problems in 2D (eg things being thrown, launched or dropped).

The most important thing to remember when dealing with questions involving 2D motion is that the vertical and horizontal components of the motion must be treated separately.

Questions involving projectiles are common exam questions, and are all essentially the same once the technique is learned.

Examples

1. An aircraft flying horizontally at 120 ms^{-1} at an altitude of 2000 metres releases an aerodynamically perfect package of food aid. Calculate:

   (a) How long the package is in the air
   (b) How far the package travels horizontally

   Start by thinking about what's going on:

   Vertically
   - The initial speed u_V is zero
   - The acceleration a_V is equal to g

   Horizontally
   - The initial speed u_H is 120 ms^{-1} and is constant
   - The acceleration a_H is zero

   (a) The time the package spends in the air is determined by its height (vertical distance) above the ground. When the package lands, its height is zero, therefore its displacement is 2000 m from when it starts falling.

   $$u_V = 0 \text{ ms}^{-1}$$
   $$a_V = 9.8 \text{ ms}^{-2}$$
   $$s_V = 2000 \text{ m}$$
   $$t = ? \text{ s}$$

   $$s = ut + \tfrac{1}{2}at^2$$
   $$s_V = \tfrac{1}{2}a_V t^2$$
   $$t^2 = \frac{2s_V}{a_V}$$
   $$t = \sqrt{\frac{2 \times 2000}{9.8}}$$
   $$= 20.2 \text{ s}$$

   So the package is in the air for 20 seconds (to the nearest second)

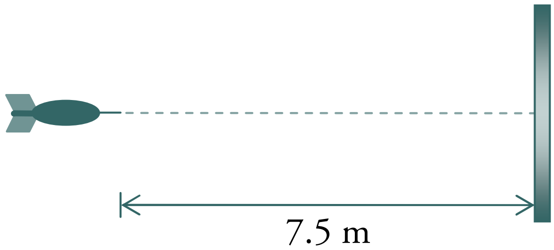
   (b) The horizontal distance travelled by the package depends on the time that it spends in the air. Since the horizontal speed can be considered constant, it is relatively simple to calculate the horizontal distance.

   $$u_H = 120 \text{ ms}^{-1}$$
   $$a_H = 9.8 \text{ ms}^{-2}$$
   $$t = 20 \text{ s}$$
   $$s_H = ? \text{ m}$$

   $$\text{distance} = \text{speed} \times \text{time}$$
   $$s_H = u_H t$$
   $$s_H = 120 \times 20$$
   $$s_H = 2400 \text{ m}$$

   So the package travels 2.4 kilometres horizontally

2. A dart is fired horizontally from a blowpipe at a speed of 26 ms^{-1} towards the centre of a target 7.5 metres away.



Calculate the distance between the centre of the target and the point of impact.

Start by thinking about what's going on:

Vertically
- The initial speed u_V is zero
- The acceleration a_V is equal to g

Horizontally
- The initial speed u_H is 26 ms^{-1} and is constant
- The acceleration a_H is zero

First, let's calculate the time taken for the dart to reach the target. We can do this by considering the horizontal motion only:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$
$$t = \frac{7.5}{26}$$
$$t = 0.29 \text{ s}$$

Now we can calculate the vertical distance (height) that the dart drops in this time period by considering only the vertical motion:

$$u_V = 0 \text{ ms}^{-1}$$
$$a_V = 9.8 \text{ ms}^{-2}$$
$$t = 0.29 \text{ s}$$
$$s_V = ? \text{ m}$$

$$s = ut + \tfrac{1}{2}at^2$$
$$s_V = \tfrac{1}{2}a_V t^2$$
$$s_V = \tfrac{1}{2}\times 9.8 \times 0.29^2$$
$$s_V = 0.41 \text{ m (to 2 s.f.)}$$

So the distance between the centre of the target and the point of impact is 41 centimetres