Higher

Mathematics

UNIT 3

Specimen NAB Assessment

HSN23510

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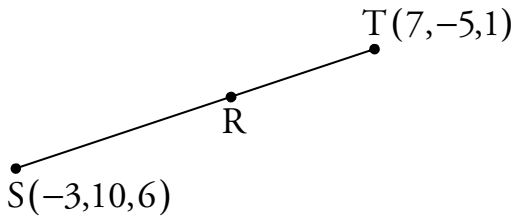
Outcome 1

1. (a) Points A, B and C have coordinates $(-4,-3,1)$, $(0,-1,0)$ and $(4,1,-1)$ respectively.

   (i) Write down the components of $\overrightarrow{AC}$.

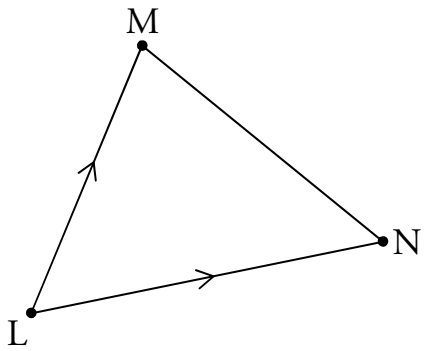
   (ii) Hence show that the points A, B and C are collinear. **4**

   (b) The point R divides $\overrightarrow{ST}$ in the ratio $3:2$, as shown below.



   Find the coordinates of R. **3**

2. The diagram shows triangle LMN where $\overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ and $\overrightarrow{LN} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$.



   (a) Find the value of $\overrightarrow{LM}.\overrightarrow{LN}$. **1**

   (b) Use your answer from part (a) to find the size of angle $M\hat{L}N$. **4**

Outcome 2

3. (a) Differentiate $-2\sin x$ with respect to x. **1**

   (b) Given $y = 5\cos x$, find $\frac{dy}{dx}$. **1**

4. Find $f'(x)$ when $f(x) = (2x+7)^{\frac{1}{3}}$. **2**

5. (a) Find $\int \left(\frac{\sqrt{3}}{2}\cos x\right) dx$. **2**

   (b) Integrate $3\sin x$ with respect to x. **1**

   (c) Evaluate $\int_4^6 (x-3)^3 \, dx$. **4**

Outcome 3

6. (a) Simplify $\log_a 7 + \log_a 3$. **1**

   (b) Simplify $\log_3 5 - 3\log_3 2$. **3**

   (c) Evaluate $\log_2 2$. **1**

7. (a) Given $x = \frac{\log_e 7}{\log_e 4}$, find an approximation for x. **1**

   (b) Given $\log_{10} y = 3{\cdot}1$, write an expression for the exact value of y. **1**

   (c) Given $y = 10^{2{\cdot}9}$, find an approximation for y. **1**

Outcome 4

8. Express $12\cos x° + 5\sin x°$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \le a \le 360$. **5**

Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{9}{12}$	$\frac{8}{11}$	$\frac{5}{8}$	$\frac{3}{5}$

Outcome 1 – Vectors	
1. (a) (i) $\overrightarrow{AC} = \boldsymbol{c} - \boldsymbol{a} = \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} ✓ = 2\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$	• Components of $\overrightarrow{AC}$ **1**
(ii) $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} ✓ = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} ✓$ Since $2\overrightarrow{AB} = \overrightarrow{AC}$ and A is a common point, A, B and C are collinear. ✓	• Know to find $\overrightarrow{AB}$ or $\overrightarrow{BC}$ • Components of $\overrightarrow{AB}$ or $\overrightarrow{BC}$ • Conclusion **3**
(b) $\frac{SR}{RT} = \frac{3}{2} ✓$ $2\overrightarrow{SR} = 3\overrightarrow{RT}$ $2(\boldsymbol{r} - \boldsymbol{s}) = 3(\boldsymbol{t} - \boldsymbol{r})$ $5\boldsymbol{r} = 3\boldsymbol{t} + 2\boldsymbol{s} ✓$ $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$ R is the point $(3,1,3).$ ✓	• Strategy for finding R • Process • State the coordinates of R **3**
2. (a) $\overrightarrow{LM}.\overrightarrow{LN} = (3 \times -2) + (4 \times 4) + (2 \times 5) = 20$ ✓	• Calculate scalar product **1**

(b) $\cos \hat{MLN} = \dfrac{\overrightarrow{LM}.\overrightarrow{LN}}{\lvert\overrightarrow{LM}\rvert\lvert\overrightarrow{LN}\rvert}$ ✓ $= \dfrac{20}{\sqrt{3^2+4^2+2^2}\sqrt{(-2)^2+4^2+5^2}}$ ✓ $= 0.554$ ✓ $\hat{MLN} = 56.4°$ ✓ (to 1 d.p.)	• Use $\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ • Start to process • Complete process • State angle	**4**
Outcome 2 – Further Calculus		
3. (a) $\frac{d}{dx}(-2\sin x) = -2\cos x$ ✓	• Differentiate	**1**
(b) $\dfrac{dy}{dx} = -5\sin x$ ✓	• Differentiate	**1**
4. $f'(x) = \frac{1}{3}(2x+7)^{-\frac{2}{3}}$ ✓×2✓ $= \dfrac{2}{3\sqrt[3]{(2x+7)^2}}$	• Differentiate term with fractional power • Use chain rule	**2**
5. (a) $\int \left(\frac{\sqrt{3}}{2}\cos x\right) dx = \frac{\sqrt{3}}{2}\sin x$✓$+c$✓	• Integrate • Constant of integration	**2**
(b) $\int (3\sin x)\, dx = -3\cos x$✓$+c$	• Integrate	**1**
(c) $\int_4^6 (x-3)^3\, dx = \left[\dfrac{(x-3)^4}{4}\text{✓}\right]_4^6$ $= \frac{1}{4}(6-3)^4 - \frac{1}{4}(4-3)^4$✓ $= \dfrac{80}{4}$ $= 20$✓	• Raise power • Correct multiplier • Substitute limits • Process	**4**
Outcome 3 – Exponentials and Logarithms		
6. (a) $\log_a 7 + \log_a 3 = \log_a 21$ ✓	• $\log_a x + \log_a y = \log_a xy$	**1**
(b) $\log_3 5 - 3\log_3 2 = \log_3 5 - \log_3 2^3$ ✓ $= \log_3 \dfrac{5}{2^3}$ ✓ $= \log_3 \frac{5}{8}$ ✓	• $k\log_a x + \log_a x^k$ • $\log_a x - \log_a y = \log_a \frac{x}{y}$ • Complete	**3**
(c) $\log_2 2 = 1$✓	• Know that $\log_a a = 1$	**1**
7. (a) 1·404 ✓ (to 3 d.p.)	• Process	**1**

(b) $y = 10^{3\cdot1}$ ✓	• Use $\log_a y = x \Leftrightarrow y = a^x$	**1**
(c) 794·3 ✓ (to 1 d.p.)	• Process	**1**
Outcome 4 – Wave Functions		
8. $k\cos(x° - a°) = k\cos a°\cos x° + k\sin a°\sin x°$ ✓ $\left.\begin{matrix} k\cos a° = 12 \\ k\sin a° = 5 \end{matrix}\right\}$ ✓ $\Rightarrow k = \sqrt{12^2 + 5^2} = 13$ ✓ $\tan a° = \frac{5}{12}$ ✓ $a = 22\cdot6$ ✓ (to 1 d.p.) So $12\cos x° + 5\sin x° = 13\cos(x° - 22\cdot6°)$.	• Use compound angle formula • Extract $k\cos a°$ and $k\sin a°$ • Calculate k • State $\tan a°$ • Calculate a	**5**