Higher

Mathematics

UNIT 1

Specimen NAB Assessment

HSN21510

UNIT 1

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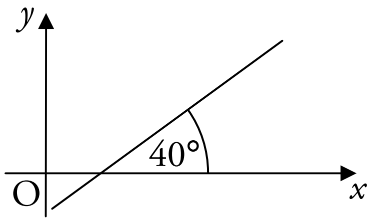
Outcome 1

1. A line passes through the points $A(4,-3)$ and $B(-6,2)$.

   Find the equation of this line. **3**

2. A line makes an angle of $40°$ with the positive direction of the x-axis, as shown in the diagram.



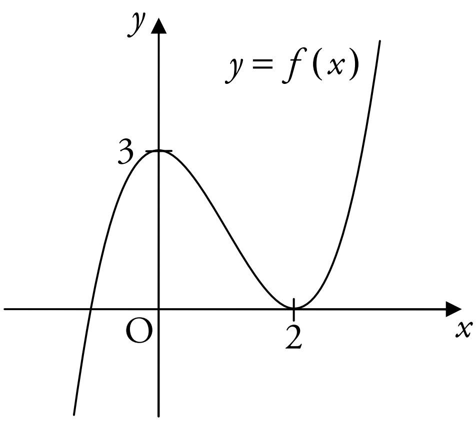
   Find the gradient of this line. **1**

3. (a) Write down the gradient of a line parallel to $y = 4x + 1$. **1**

   (b) Write down the gradient of a line perpendicular to $y = 4x + 1$. **1**

Outcome 2

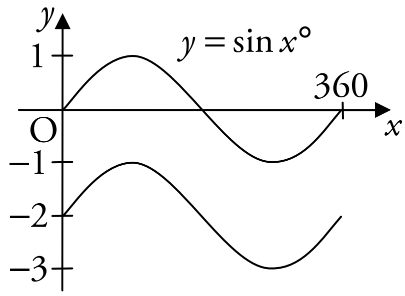
4. The diagram below shows part of the graph of $y = f(x)$.



   (a) Sketch the graph of $y = -f(x)$. **1**

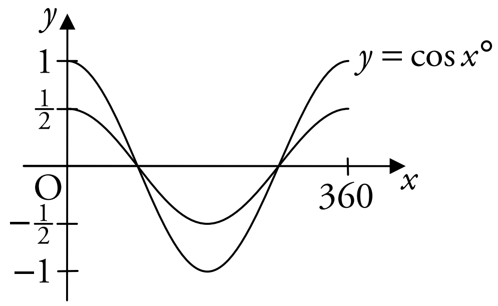
   (b) On a separate diagram, sketch the graph of $y = f(x+4)$. **1**

5. (a) The diagram below shows the curve $y = \sin x°$ and a related curve.



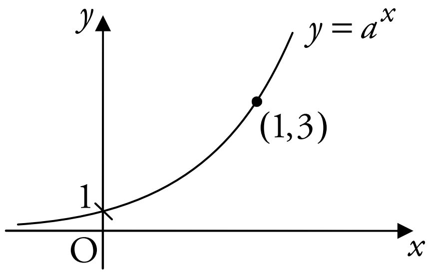
Write down the equation of the related curve. **1**

(b) The diagram below shows the curve $y = \cos x°$ and a related curve.



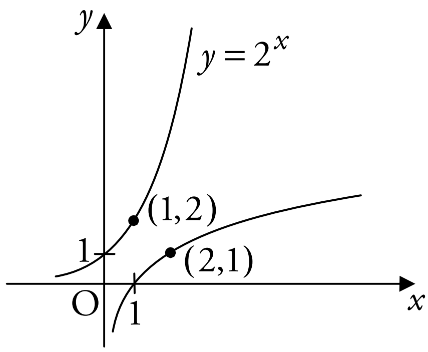
Write down the equation of the related curve. **1**

6. The curve $y = a^x$ is shown in the diagram below.



Given that the curve passes through the point $(1,3)$, write down the value of a. **1**

7. The diagram below shows the graph of the function $f(x) = 2^x$ and its inverse function.



Write down the formula for the inverse function. **1**

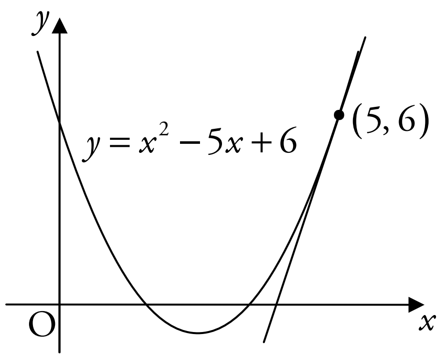
8. (a) Two functions f and g are defined by $f(x)=x^3$ and $g(x)=2x-4$. Find an expression for $f(g(x))$. **1**

(b) Functions h and k are defined on suitable domains by $h(x)=5x$ and $k(x)=\tan x$.
Find an expression for $k(h(x))$. **1**

Outcome 3

9. Given that $y=\dfrac{x^5-3}{x^3}$ for $x\neq 0$, find $\dfrac{dy}{dx}$. **4**

10. The curve with equation $y=x^2-5x+6$ is shown below.



Find the gradient of the tangent to the curve at the point $(5,6)$. **4**

11. A curve has equation $y=\frac{1}{3}x^3-4x^2+12x-3$.
Find the stationary points on the curve and, using differentiation, determine their nature. **8**

Outcome 4

12. A pond is treated weekly with a chemical to ensure that the number of bacteria is kept low. It is estimated that the chemical kills 68% of all bacteria. Between the weekly treatments, it is estimated that 600 million new bacteria appear. There are u_n million bacteria at the start of a particular week.

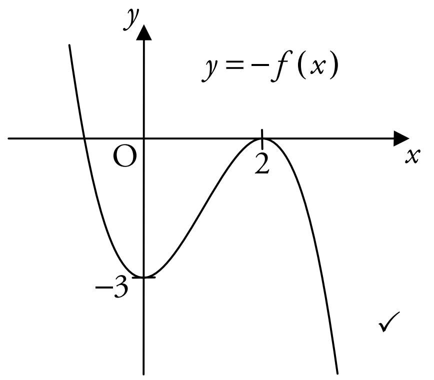
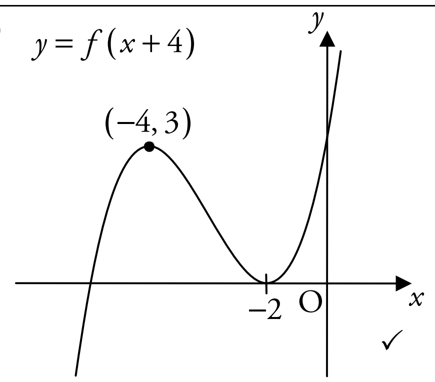
(a) Write down a recurrence relation for u_{n+1}, the number of millions of bacteria at the start of the next week. **1**

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question. **3**

Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{4}{6}$	$\frac{6}{8}$	$\frac{11}{16}$	$\frac{3}{4}$

Outcome 1 – Straight Lines		
1. $m = \frac{2-(-3)}{-6-4}$ ✓ $= \frac{5}{-10}$ $= -\frac{1}{2}$ ✓ $y - 2 = -\frac{1}{2}(x+6)$ ✓ $2y - 4 = -x - 6$ $x + 2y + 2 = 0$	• Use gradient formula • Calculate gradient • Equation of line	3
2. $m = \tan 40°$ $= 0{\cdot}84$ (to 2 d.p.) ✓	• Calculate gradient	1
3. (a) 4 ✓	• State gradient	1
(b) $-\frac{1}{4}$ ✓	• State gradient	1
Outcome 2 – Functions and Graphs		
4. (a) 	• Sketch showing images of given points	1
(b) 	• Sketch showing images of given points	1

5. (a) $y = \sin x° - 2$ ✓	• Identify equation	1
(b) $y = \frac{1}{2}\cos x°$ ✓	• Identify equation	1
6. Since $y = 3$ when $x = 1$: $a^1 = 3$ $a = 3$ ✓	• State the value of a	1
7. $f^{-1}(x) = \log_2 x$ ✓	• State formula for inverse	1
8. (a) $f(g(x)) = f(2x-4)$ $= (2x-4)^3$ ✓	• Expression for composite function	1
(b) $k(h(x)) = k(5x)$ $= \tan 5x$ ✓	• Expression for composite function	1
Outcome 3 – Differentiation		
9. $y = \frac{x^5}{x^3} - \frac{3}{x^3}$ $= x^2$✓$-3x^{-3}$✓ $\frac{dy}{dx} = 2x$✓$+9x^{-4}$✓	• Simplify first term • Simplify second term • Differentiate first term • Differentiate second term	4
10. Gradient of tangent is given by $\frac{dy}{dx}$ ✓ $\frac{dy}{dx} = 2x - 5$ ✓ At $x = 5$✓, $m = 2 \times 5 - 5$ $= 5$ ✓	• Know to differentiate • Differentiate • Know to evaluate derivative • Calculate gradient	4

11. $\frac{dy}{dx}$ ✓ $= x^2 - 8x + 12$ ✓ Stationary points exist where $\frac{dy}{dx} = 0$ $x^2 - 8x + 12 = 0$ ✓ $(x-6)(x-2) = 0$ $x = 2$ or $x = 6$ ✓ To find y-coordinates: At $x = 6$, $y = \frac{1}{3}(6)^3 - 4(6)^2 + 12(6) - 3$ $= -3$ At $x = 2$, $y = \frac{1}{3}(2)^3 - 4(2)^2 + 12(2) - 3$ $= 7\frac{2}{3}$ ✓ Stationary points are at $\left(2, 7\frac{2}{3}\right)$ and $(6, -3)$ $\begin{array}{c\|ccccc} x & \rightarrow & 2 & \rightarrow & 6 & \rightarrow \\ \hline \frac{dy}{dx} & + & 0 & - & 0 & + \\ \text{sketch} & / & - & \backslash & \_ & / \end{array}$ ✓ $\left(2, 7\frac{2}{3}\right)$ is a maximum turning point ✓ $(6, -3)$ is a minimum turning point ✓	• Know to differentiate • Differentiate • Set derivative equal to 0 • Find x-coordinates of stationary points • Find y-coordinates of stationary points • Method to determine nature • Nature of one stationary point • Nature of second stationary point	**8**
Outcome 4 – Sequences		
12. (a) $u_{n+1} = 0{\cdot}32u_n + 600$	• State recurrence relation	**1**
(b) A limit l exists since $-1 < 0{\cdot}32 < 1$ $l = \frac{600}{1-0.32}$ ✓ $= 882{\cdot}35$ ✓ (to 2 d.p.) In the long term, the number of bacteria will settle around 882 million ✓	• Know how to calculate limit • Calculate limit • Interpret limit	**3**